Homework 12

P11.1.5 The switch in Figure P11.1.5 is moved to position 'b' at $t = 0$ after being in position 'a' for a long time. Determine, for $t \geq 0^+$: (a) $v_C(t)$; (b) $v_O(t)$; (c) $i_O(t)$; (d) the total energy dissipated in the 60 kΩ resistor as $t \to \infty$.

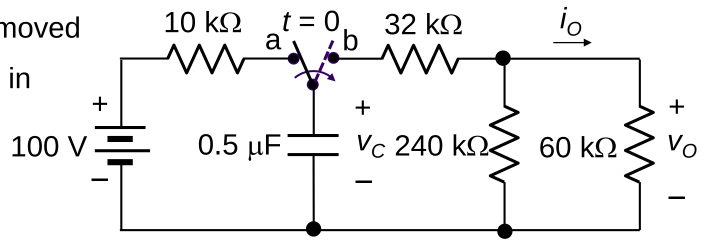

Figure P11.1.5

Solution: (a) $v(0^+) = 100$ V; $V_F = 0$; the resistance seen by C is: 32 + 240||60 = 32 + 48 = 80 kΩ; $\tau = 0.5\times10^{-6}\times80\times10^3 = 0.04$ s; it follows that $v_C(t) = 100\,e^{-25t}$ V, t is in s.

(b) From voltage division, $v_O(t) = (48/80)v_C(t) = 60e^{-25t}$ V.

(c) $i_O(t) = v_O(t)/60 = e^{-25t}$ mA.

(d) The total energy dissipated in the 60 kΩ resistor is $60\times10^3\int_0^\infty i_O^2(t)dt =$

$60\times10^{-3}\int_0^\infty e^{-50t}dt = \frac{60}{50}\times10^{-3} \equiv 1.2$ mJ. Alternatively, The initial energy stored in the capacitor is $0.5\times0.5\times10^{-6}\times10^4 \equiv 2.5$ mJ; the energy dissipated in the 48 kΩ resistor in series with the 32 kΩ resistor is 2.5×48/80 = 1.5 mJ; this energy is dissipated in 60 kΩ in parallel with 240 kΩ, and divides in proportion to the conductances. The energy dissipated in the 60 kΩ resistor is therefore

$$\frac{1/60}{1/60+1/240}\times1.5 = \frac{4}{5}\times1.5 = 1.2 \text{ mJ.}$$

P11.1.6 The switch in Figure P11.1.6 is opened at $t = 0$ after being closed for a long time. Determine $v_O(t)$ for $t \geq 0^+$.

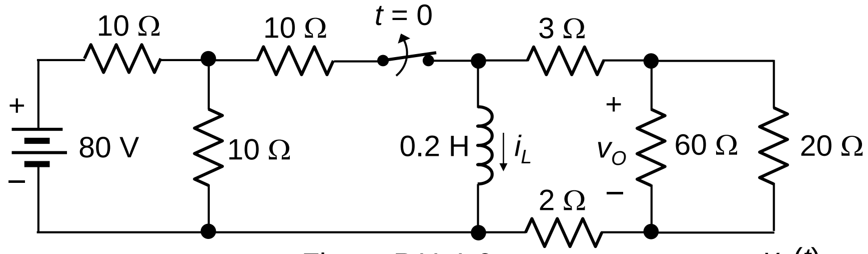

Figure P11.1.6

Solution: Before the switch is opened, the inductor behaves as a short circuit; the source current is 80/15 = 16/3 A, so that $i_L(0^+) = (16/3)/2 = 8/3$ A; $I_{LF} = 0$; the resistance seen by inductor is 5 + 60||20 = 20 Ω; $\tau = 0.2/20 = 0.01$ s. It follows that $i_L(t) = (8/3)e^{-100t}$ A, t is in s; from current division, $v_O(t) = -15i_L(t) = -40e^{-100t}$ V, t is in s.

P11.1.10 The switch in Figure P11.1.10 Is opened at $t = 0$ after being closed for a long time. Determine $i_X(t)$, $t \geq 0^+$.

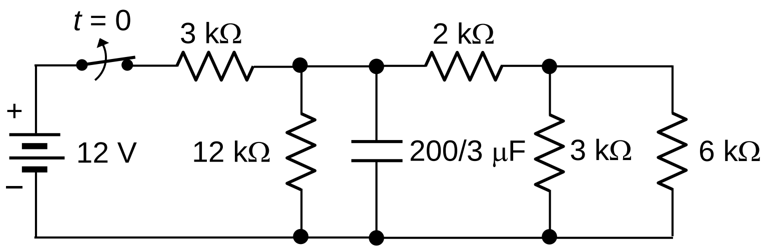

Figure P11.1.10

Solution: After the switch has been closed for a long time, the capacitor behaves as an open circuit; 6||3 = 2 kΩ; in series with 2 kΩ, this is 4 kΩ; in parallel with 12 kΩ, this is 3 kΩ; from voltage division, the initial voltage on the capacitor is 6 V; the current through the 2 kΩ resistor is 6/4 = 1.5 mA, the voltage across the parallel combination is 3 V, and I_X = 1 mA. When the switch is opened the capacitor voltage does not change at this instant, so I_X does not change. Hence, $i_X(0^+) = 1$ mA; $I_{XF} = 0$; the resistance seen by the capacitor after the switch is opened is 3 kΩ; $\tau = (200/3)\times10^{-6}\times3\times10^3 = 0.2$ s. It follows that $i_X(t) = e^{-5t}$ mA, t is in s.

P11.1.12 Both switches In Figure P11.1.12 have been closed for a long time. The first switch opens at $t = 0$ and the second switch opens at $t = 35$ ms.

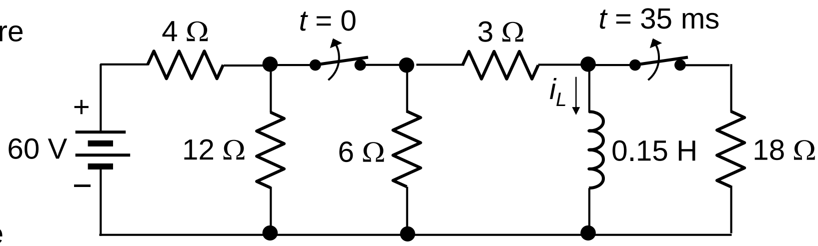

Figure P11.1.12

Determine: (a) $i_L(t)$ for $0 \leq t \leq 35$ ms; (b) $i_L(t)$ for $t \geq 35$ ms; (c) the percentage of the energy initially stored in the inductor that is dissipated in the 18 Ω resistor.

Solution: (a) When the switches have been closed for a long time, the inductor behaves as a short circuit, To determine the initial value of i_L, the voltage source in series with 4 Ω can be transformed to a current source of 15 A in parallel with 4 Ω. It follows from current division that $i_L(0^+) = 15\frac{1/3}{1/4 + 1/12 + 1/6 + 1/3} = 6$.

Between $t = 0$ and $t = 35$ ms, the resistance seen by the inductor is18||9 = 6 Ω; hence, $\tau = 0.15/6 = 0.025$ s ≡ 25 ms. It follows that $i_L(t) = 6e^{-t/25}$ A, $0 \leq t \leq$ 35 ms

(b) At $t = 35$ ms, $i_L(35) = 6e^{-35/25} = 1.48$ A; the resistance seen by the inductor after both switches are open is 9 Ω, so that $\tau = 0.15\times9 \equiv 50/3$ ms. It follows that $i_L = 1.48e^{-3(t-35)/50}$, $t \geq 35$ ms

(c) For $0 \leq t \leq 35$ ms, the current in the 18 Ω resistor is $i_L/3 = 2e^{-t/0.025}$ A, t is in s;

the energy dissipated during this period is $18\times 4\int_0^{0.035} e^{-2t/0.025}dt = 0.8453$ J.

The initial energy stored in the inductor is $0.5\times 0.15\times (6)^2 = 2.7$ J.

The percentage is $(0.8435/2.7)\times 100 = 31.31\%$.

P11.1.13 The switch in Figure P11.1.13 is moved to position 'b' at t = 0, after being in position 'a' for a long time. Determine $v_C(t)$ for $t \geq 0^+$.

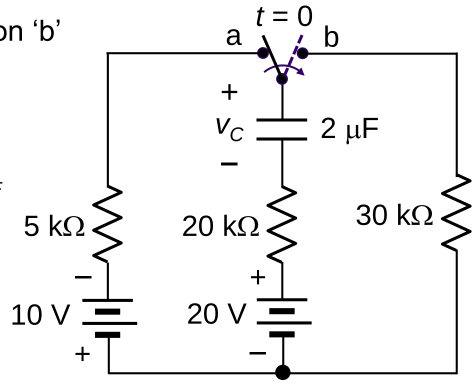

Figure P11.1.13

Solution: Just before the switch is moved, $V_{C0} = -30$ V; $V_{CF} = -20$ V; $\tau = 50\times 10^3 \times 2\times 10^{-6} = 0.1$ s. Hence,